DESIGN FOR SEISMIC TORSIONAL FORCES

by

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ABSTRACT

An analytical study of the response of a single storey mono-symmetric building model to a combined torsional and rotational ground motion is presented. The ground excitations are represented by idealized spectra and the building model is assumed to be elastic.

A code provision for design eccentricity is proposed. The forces obtained by the use of the proposed method are compared with the analytical results obtained from the single storey model and are shown to provide an adequate design basis.

INTRODUCTION

Buildings subjected to earthquakes often undergo a torsional motion in addition to the translational motions in three orthogonal directions. The torsional motion may arise due to an eccentricity between the centres of mass and resistance at various floors of the building. A more direct cause of torsion, one which will excite a torsional response even in a symmetric building, is the rotational component of ground motion about a vertical axis.

Most seismic codes make special provisions to allow for the forces arising from torsional oscillations. In particular, the National Building Code of Canada (NBCC) specifies that the torsional moments be obtained by assuming that at any level of the building the resultant seismic force acts through a point which is eccentric with respect to the centre of resistance at that level. The design eccentricity e_x is given by

$$e_x = 1.5e + 0.05D \tag{1}$$

in which e is the distance between the centres of mass and resistance and D is a specified plan dimension at the level under consideration. The factor 1.5 is applied to the estimated eccentricity e to account for possible dynamic amplification. The second term in Eq. 1 is designated as accidental eccentricity and is supposed to take care of possible difference between the actual and estimated eccentricities as well as of the torsion arising from ground rotation.

The torsional phenomenon and the validity of code provisions have been investigated by several researchers (1,2,3,4). All of these studies have used the single storey monosymmetric building model shown in Fig. 1(a). Tso and Demsey (2) have compared the magnitudes of eccentricities obtained from a dynamic analysis with those specified by the NBCC 1980. On the basis of this comparison, they conclude that the code provisions underestimate the torsional moments when the eccentricity of the building is small and the ratio of uncoupled torsional frequency to the uncoupled translational frequency is close to unity. Tsicnias and Hutchinson (3) arrive at a similar conclusion.

From the foregoing, it appears there is some evidence to show that the present torsional provisions of the NBCC are non-conservative. There is also a certain amount of practical difficulty in the application of the existing code provisions to the design of multistorey buildings. This paper therefore attempts to take a closer look at the torsional provisions. Much of the present work is based on an elastic analysis of the single storey building model of Fig. 1. It is however believed that the results will also apply to many multistorey buildings. It is shown that the present code provisions are not necessarily unconservative but that they can be somewhat simplified.

EVALUATION OF CODE PROVISIONS

It is desirable that any code provisions be reasonably simple to apply and be rational at the same time. If the torsion provisions of the NBCC are viewed in light of the foregoing criteria, following important considerations emerge.

1. In a building it is reasonably simple to estimate the position of the centre of mass. Therefore, if the requirement is to apply the seismic forces either through the centre of mass or through a point away from the centre of mass at a distance which is a linear function of the plan dimension, the analysis of the structure is quite straightforward. On the other hand, if the point of application of the seismic forces is specified to be at a certain distance from the centre of resistance, which is a function of the eccentricity e, then the value of this eccentricity and hence the position of the centre of resistance must be determined.

For multistorey buildings, there does not appear to be any accepted definition for the centre of resistance at a floor. It is possible to define the centre of resistance at any level as a point such that when a lateral force is applied through it, the level under consideration does not undergo any rotation (other levels may twist). Even when this definition is accepted, the actual determination of the centres of resistance at all floors of a real multistorey building may be quite complex and time consuming. In practice added difficulty arises from the fact that most commercially available two or three dimensional frame analysis programs do not have any simple mechanism for determining the locations of the centres of resistance.

As an example, consider the ten storey building shown in Fig. 2. A rather complicated analysis taking the interaction of frames and shear walls into account is required to locate the centres of resistance whose positions are shown in the figure. It will be noted that centres of resistance do not lie on a vertical line, eventhough the frames and shearwalls have a uniform section throughout the height. An even more dramatic illustration of the variation in the location of the centre of resistance is provided by the building shown in Fig. 3., where the eccentricity varies from - 143.7 in. at the 2nd floor level to +104.3 in. at the roof level even as the frame and shearwall sections remain uniform.

2. Studies on the effect of torsional ground motion (4,5) have shown that often such effects may be more important than those due to dynamic amplification caused by coupling. This is particularly so when the ratio of the uncoupled torsional frequency to the uncoupled translational frequency is substantially greater than 1. Also, the contribution of ground torque takes added significance when the plan eccentricity is small.

3. From the point of view of design for seismic forces, the maximum force induced in a lateral resisting element is of greater significance than the maximum values of the storey shears and torques. Consider the single storey model of Fig.1. The force on a lateral resisting element can be obtained by the application of the maximum seismic shear through the centre of resistance together with the maximum seismic torque about the same point. A more direct method of obtaining such force would be to obtain the maximum displacement of the resisting element and to multiply this displacement by the stiffness of the stiffness. Because of the dynamic nature of the problem, the results obtained by the two methods will not be identical.

The foregoing is best explained by considering an example, say of the model shown in Fig.1. Figure 1b shows the two vibration modes of the model. It will be noted that for element 'a', which is farthest from the centre of resistance, the lateral displacement in the second mode due to torsion counteracts that due to shear. Therefore to obtain the resultant displacement of 'a' the displacements in each of the two modes should be separately evaluated and then superposed. The alternative method of obtaining the resultant torque and shear and then using them to obtain the displacement will give an exaggerated value.

In the remaining part of this paper, a detailed analysis is presented of the building model of Fig. 1. An alternative torsion provision which fulfils some of the criteria mentioned in the foregoing paragraphs is presented and the design eccentricities and edge displacements derived from the proposed method are compared with their values obtained from analyses.

ANALYSIS PROCEDURE

The single storey building model shown in Fig.1 consists of a rectangular rigid deck supported on axially rigid columns and/or walls. The mass and stiffness are so distributed that the model has one axis of symmetry as shown. In the analysis presented here, two degrees of freedom are considered: translation v normal to the direction of symmetry, and the rotation θ about a vertical axis, both considered at the centre of resistance and measured relative to the ground. The third degree of freedom, translation parallel to the axis of symmetry, is uncoupled from the first two, and can be considered seperately if desired.

The undamped equations of motion of the model are given by

$$\begin{bmatrix} \Omega^2 & \Omega e^* \\ \Omega e^* & 1 \end{bmatrix} \begin{Bmatrix} \ddot{v} + \ddot{v}_g - e\ddot{\theta}_g \\ \ddot{\psi} \quad \ddot{\psi}_g \end{Bmatrix} + \omega_\theta^{\,2} \begin{Bmatrix} v \\ \psi \end{Bmatrix} = 0 \qquad (2)$$

in which, m = the mass of the deck, e = the static eccentricity, ρ= the radius of gyration of the deck about the centre of resistance, k_v = the translational stiffness, k_θ = the torsional stiffness about the centre of resistance, $\ddot{v}_g$ = the translational ground acceleration and $\ddot{\theta}_g$ = the torsional ground acceleration both measured about the centre of mass, $\omega_v = (k_v/m)^{1/2}$, the uncoupled translational frequency, $\omega_\theta = (k_\theta/m\rho^2)^{1/2}$, the uncoupled torsional frequency, $\Omega = \omega_\theta/\omega_v$, $e^* = e/\rho$, $\ddot{\psi} = \Omega\rho\ddot{\theta}$ and $\ddot{\psi}_g = \Omega\rho\ddot{\theta}_g$.

The equations of motion (Eq.2) can be solved by the mode superposition method. The natural frequencies, ω_n, and the mode shapes, $\{\alpha_n\}$, of the system required for a modal solution are obtained by solving Eq. 2 with $\ddot{v}_g$ and $\ddot{\theta}_g$ each equal to zero. The results can be shown to be

$$\omega_n = \frac{\omega_v \, \Omega}{\left[\frac{1+\Omega^2}{2} - (-1)^n R\right]^{1/2}} \qquad n = 1,2 \qquad (3)$$

in which

$$R = \sqrt{\left(\frac{1-\Omega^2}{2}\right)^2 + e^{*2}\,\Omega^2} \qquad (4)$$

and

$$\alpha_1 = \begin{Bmatrix} \alpha_{x1} \\ \alpha_{\theta 1} \end{Bmatrix} = \begin{Bmatrix} \cos\phi \\ \sin\phi \end{Bmatrix} \qquad \alpha_2 = \begin{Bmatrix} \alpha_{x2} \\ \alpha_{\theta 2} \end{Bmatrix} = \begin{Bmatrix} -\sin\phi \\ \cos\phi \end{Bmatrix} \qquad (5)$$

where $\tan 2\phi = \dfrac{2\Omega e^*}{\Omega^2 - 1}$ (6)

The modal equations are obtained by introducing the following transformation in Eq. 3

$$\begin{Bmatrix} v \\ \psi \end{Bmatrix} = \sum_{n=1}^{2} Y_n \{\alpha_n\} \qquad (7)$$

where Y_n are the modal coordinates. Damping is intoduced at this stage directly into the modal equations by defining appropriate viscous damping factors.

The maximum values of the modal coordinates Y_n can be obtained directly from response spectra. For ground translation two different types of spectral shapes are considered: a flat response spectrum in which the spectral acceleration does not vary with ω_n, and a hoperbolic response spectrum in which the acceleration varies directly as ω_n or inversely as the period. Response spectra for torsional ground motion are derived from the translational spectra by a procedure suggested by Newmark (4,5).

The response quantities of interest in the present study are the base shear V normal to the direction of symmetry, the base torque T at the centre of resistance and the displacement Δ of the edge of the deck farthest from the centre of resistance.

The maximum value of a response quantity Q in each natural mode of vibration can be obtained from the maximum values of the corresponding modal coordinates by using an appropriate transformation. The modal maxima Q_n are then combined according to the following equation (1).

$$Q^2 = \sum_{n=1}^{2} Q_n^2 + \sum_{n=1}^{2} \sum_{\substack{m=1 \\ n \neq m}}^{2} \frac{Q_n Q_m}{1+\varepsilon_{mn}^2} \tag{8}$$

in which

$$\varepsilon_n = \frac{\sqrt{1-\xi^2}}{\xi} \frac{\omega_n - \omega_m}{\omega_n + \omega_m} \tag{9}$$

and the damping factor ξ is considered to be the same in each mode of vibration.

For the presentation of results, it is convenient to normalize the modal maxima for the base shear by the base shear, $V_o = mS_{av}(\omega_n, \xi)$, of the uncoupled system. The torques are normalized by $mrS_{av}(\omega_n, \xi)$, in which r is the radius of gyration about the mass centre, and the displacements are normalized by $\Delta_o = V_o/k_v$. The normalized values are denoted respectively by $\bar{V}$, $\bar{T}$ and $\bar{\Delta}$.

The modal responses for any one type of excitation can be combined according to Eq. 8 to give the total response due to that excitation. If the excitation has more than one component, the responses due to individual components can be further superimposed by the root sum square combination.

RESPONSE OF SINGLE STOREY BUILDING MODEL

Analytical results were obtained for the response of the single storey building model subjected to ground motions represented by a flat and a hyperbolic translational acceleration spectrum, each with its associated torsional response spectrum (4). The damping was assumed to be 5% of the critical in each of the two modes. For brevity, the results for only the flat spectrum are presented here.

The component of response due to ground rotation depends on the time τ that a shear wave takes to travel a distance D, the plan dimension. Results are presented here for three different values of τ: 0.05s, 0.1s and 0.15s. For τ=0.1 the response values are shown for each of the two excitation components; translation and rotational. The resultant response obtained by a superposition of the two components is also shown. For other values of τ only the resultant response is shown.

Figs. 4 and 5 show the normalized dynamic eccentricity $\frac{e_d}{r} = \bar{T}$ and the normalized edge displacement $\bar{\Delta}$ plotted as functions of e/r for Ω=1 and 1.5 and three different values of τ.

Keeping in view the desirable criteria for a code provision on torsion stated previously, the following expression is proposed for design eccentricity

$$e_x = e + 0.1D \tag{10}$$

The normalized design eccentricity can be obtained directly from Eq. 10 on division by r. The normalized edge displacement that corresponds to the design eccentricity of Eq. 10 is given by

$$\bar{\Delta} = 1 + \frac{e_{x/r}(e/r + \frac{D}{2r})}{(e/r)^2+1} \tag{11}$$

Values of $e_{x/r}$ and $\bar{\Delta}$ obtained from Eqs. 10 and 11 are also shown in Figs. 4 and 5. The analytical results show that the ground torque makes a significant contribution to the total response, particularly for systems with small eccentricities. Noting that the component of response, due to translation does not vary with τ, it is apparent the effect of ground torque on response increases as the travel time of the shear wave increases.

Figs. 4a and 5a show that the design eccentricity obtained from Eq. 10 is smaller than its analytical value, at least for systems with small and moderate eccentricities. This does not however mean that the use of Eq. 10 will lead to non-conservative estimates of the design forces in lateral resisting elements because when the edge displacements calculated from Eq. 11 are compared with those obtained from dynamic analyses, as shown in Figs. 4b and 5b, an altogether different picture emerges. The design eccentricity given by Eq. 10 now appears to be quite satisfactory.

The results obtained for hyperbolic and a combination of flat and hyperbole spectra (not presented here) lead to similar conclusions.

SUMMARY AND CONCLUSIONS

Based on the analytical study of the response of a single storey building subjected to ground motion, a proposal is made for a design eccentricity which when used with the equivalent static load method of seismic design will account for the presence of torsional forces. The expression proposed for the eccentricity is somewhat similar to the one in the current Uniform Building Code. It is simple in application and does not require explicit determination of the centres of resistance.

For many multistorey buildings, the response can be shown to be governed by the response of an associated single storey model (1). The conclusions drawn here for a single storey building are therefore expected to apply to such multistorey buildings.

REFERENCES

1. Kan, C.L. and Chopra, A.K., "Coupled Lateral Torsional Response of Buildings to Ground Shaking", Report No. 76-13, University of California, Berkeley, California (1976).

2. Tso, W.K., and Dempsey, K.M., "Seismic Torsional Provisions for Dynamic Eccentricity", Earth, Engng. and Struct. Dyn., Vol. 8, pp. 275-289 (1980).

3. Tsicnias, T.G., and Hutchinson, G.L., "Evaluation of Code Requirements for the Earthquake Resistant Design of Torsionally Coupled Buildings", Proc. Instn. Engrs., Part 2, Vol. 71, pp. 821-843 (1981).

4. Awad, A.M., and Humar, J.L., "Dynamic Response of Building to Ground Motion", Seventh European Conference on Earthquake Engineering, Athens, Greece (1982).

5. Newmark, N.M., "Torsion in Symmetric Buildings", Proceedings Fourth World Conference on Earthquake Engineering, Vol. 2, Santiago, pp. A3, 19-30 (1969).

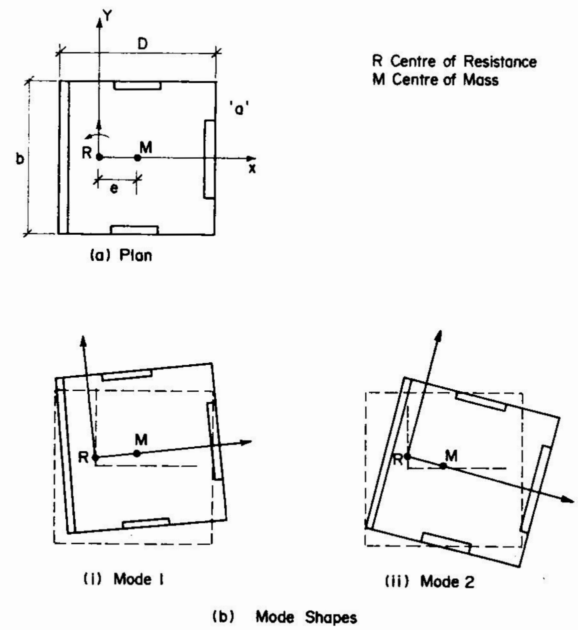


Fig.1 Single Storey Building Model

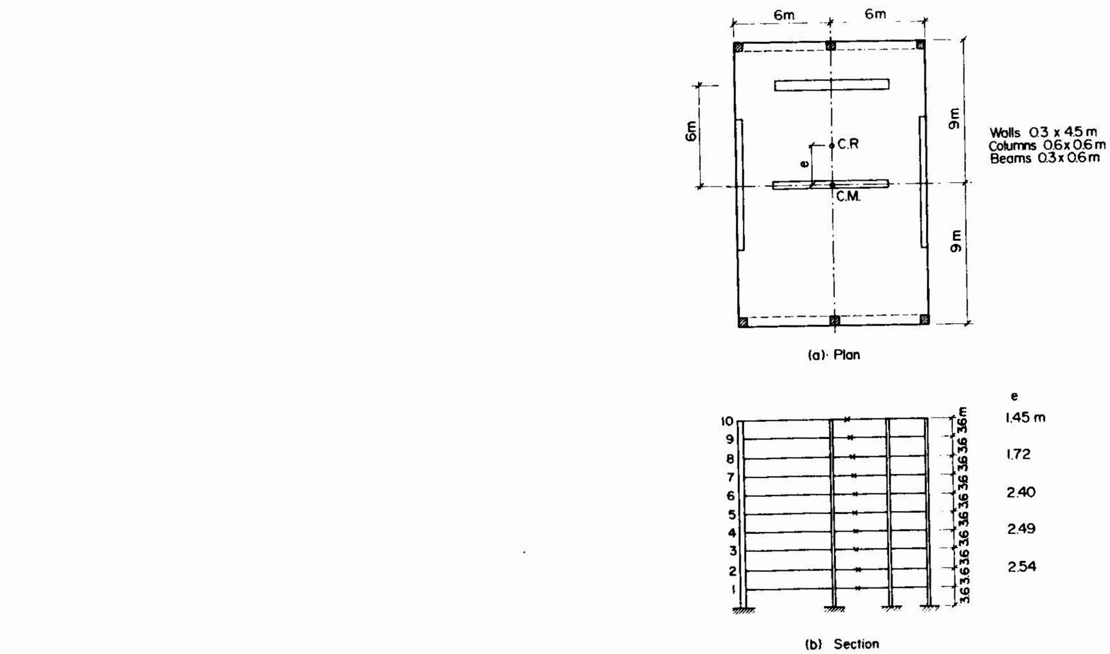


Fig.2 Ten Storey Building, Moderate e

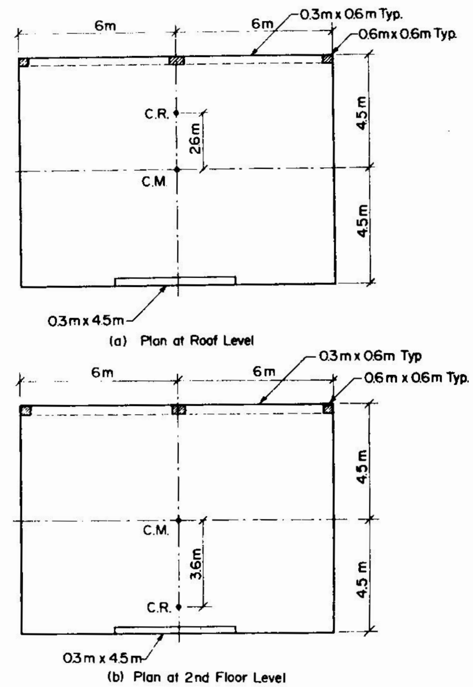
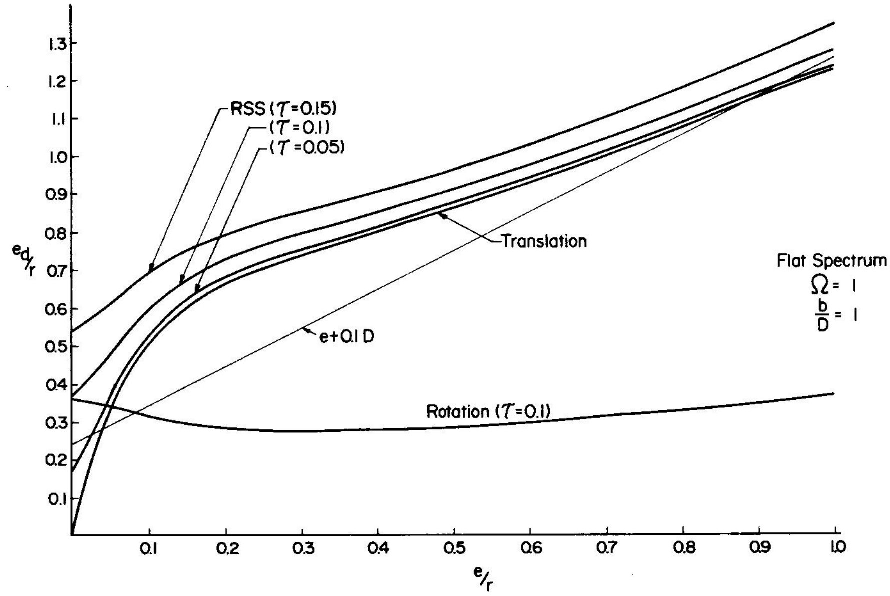
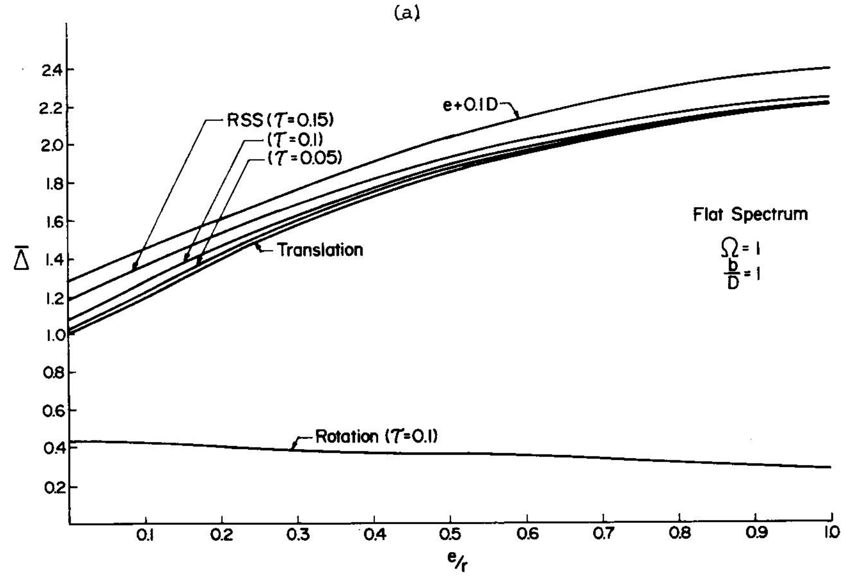


Fig.3 Ten Storey Building, Large e

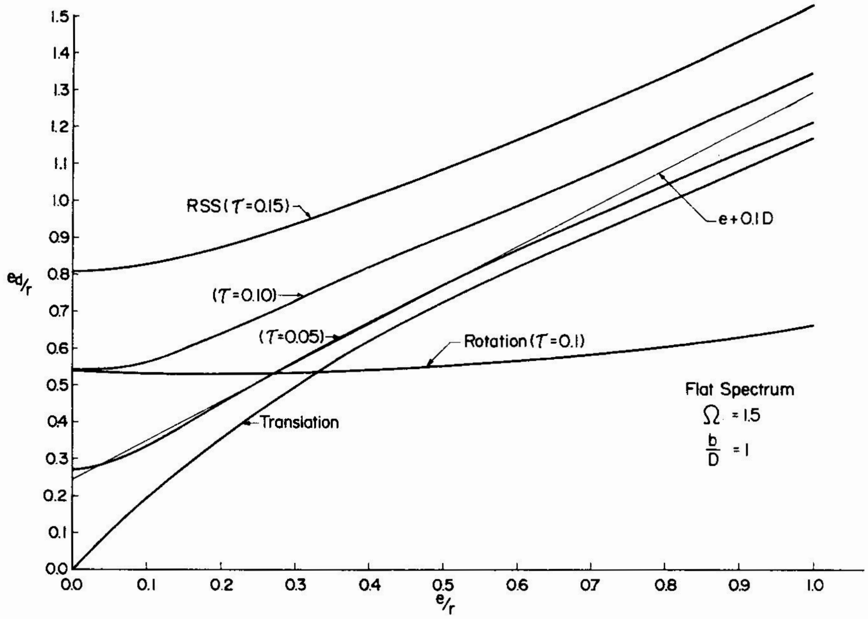


(a)

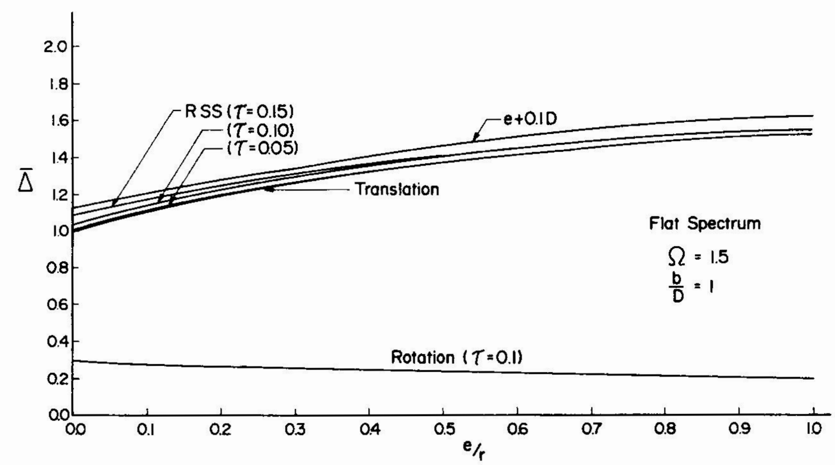


(b)

Fig.4 Response of Building Model, Frequency Ratio = 1.0



(a)



(b)

Fig.5 Response of Building Model, Frequency Ratio = 1.5